Lecture 4

Market Microstructure

Market Microstructure

- Hasbrouck: “Market microstructure is the study of trading mechanisms used for financial securities.”
- New transactions databases facilitated the study of high frequency phenomena in various markets. (Equities: TAQ; NASTRAQ. FX: Olsen; Fixed Income: TRACE, Warga)
- The majority of research has been on equities and foreign exchange, much less on fixed income.

Market Architecture

- Securities also trade in a hybrid environment of market designs or *architectures*.

- NYSE: Floor based auction organized by a specialist
- Nasdaq: Interdealer electronic network
- ECNs (ATS): Electronic networks with no dealer intermediaries (e.g. Archipelago, Instinet for equities, BrokerTec, eSpeed for U.S. Treasuries)
- Open outcry: CME, CBOT futures pits, Treasury phone based market

- This is a rich area for empirical industrial organization research.

Liquidity

- Liquidity is like pornography. Easy to identify when seen, but it is difficult to define. But, CLM defines liquidity as:

  "*Ability to buy or sell significant quantities of a security quickly, anonymously, and with minimal or no price impact.*"

- Market-makers: provide liquidity by taking the opposite side of a transaction. If an investor wants to buy, the market-maker sells and vice versa.
- In exchange for this service, market-makers buy at a low bid price P^b and sell at a higher ask price P^a: This ability insures that the market-makers will make some profits.
- The difference P^a - P^b is called the bid-ask spread. A trading cost.

• High trading costs (commissions, fees, opportunity costs, bid-ask spreads, etc.) are linked to low liquidity.

• Related concepts:

- Depth: The quantity available for sale or purchase away from the current market price.

- Breadth: The market has many participants.

- Resilience: Price impacts caused by the trading are small and quickly die out.

Roll (1984)

- The bid-ask spread complicates research, since we don't observe the true price.
  - We have three prices: bid, P^b, ask, P^a, and true price, P^*.
  - The true price is often between P^a and P^b, although it need not be.
  - How do we define returns: From P^a to P^a, P^b to P^b , P^b to P^a...?
  - How is $P^a – P^b$ determined?
- It is fairly intuitive that the bid-ask spread has an effect on returns.
- Roll (1984) provides a simple model of how the bid-ask spread might impact the time-series properties of returns.
- Roll (1984) provides most of the intuition and the framework on how financial economists think about the bid-ask spread.

• The observed market price is

$$P_t = P_t^* + q_t\, s/2 .$$

P_t^*: fundamental price in a frictionless economy

s: bid-ask spread (independent of the P_t level)

q_t: iid index variable -takes values of 1 with prob. 0.5 (buy)

-takes value of -1 with prob. 0.5 (sell).

• q_t is unobservable. But, with the assumptions, $E[q_t] = 0$ and $Var(q_t) = 1$.

• For simplicity assume that P_t^* does not change - $Var(\Delta P_t^*)=0$.

• The change in price is:

$$\Delta P_t = \Delta P_t^* + q_t\, s/2 - q_{t-1}\, s/2 = \Delta P_t^* + c\, \Delta q_t. \quad (c=s/2)$$

• Its variance, covariance, and correlation are:

$Var(\Delta P_t) = Var(\Delta P_t^*) + c^2\, Var(I_t) + c^2\, Var(I_t) = 2c^2 \;\; (= s^2/2)$

$Cov(\Delta P_t, \Delta P_{t-1}) = -c^2$

$Cov(\Delta P_t, \Delta P_{t-k}) = 0;\ k > 1$

$Corr\,(\Delta P_t, \Delta P_{t-1}) = -1/2$

• Note:
  - • The fundamental value is fixed, but there is variation from c.
  - • The bid-ask spread induces negative correlation in returns even in the absence of other fluctuations.
  - • The variance and covariance depend on the magnitude of the bid-ask spread.
  - • In this particular example, it induces a 1st-order serial correlation.

• We can also express the spread as a function of the covariance:

$c = [-Cov(\Delta P_t, \Delta P_{t-1})]^{-1/2}$

• In practice, we can find $Cov(\Delta P_t, \Delta P_{t-1})] > 0$. (Misspecification?: Glosten and Harris (1988) and Stoll (1989).)

• To avoid this problem, Roll (1984) defines the spread as

$c = -\,[|Cov(\Delta P_t, \Delta P_{t-1})|]^{-1/2}$

Roll calls s(=2c) the “effective spread,” which is estimable.

• Roll's (1984) model illustrates how the spread can induce negative serial correlation in returns. The serial correlation is a function of the spread. But, the spread is set exogenously.

• Q: What determines the bid-ask spread?
  - Order-processing costs: basic setup and operation costs.
  - Inventory costs: holding an undesired security (risk!).
  - Adverse selection costs: some investors are better informed than the market maker about the stock. Glosten (1987)

• Things to consider:
  - The spread is unlikely to be independent of P_t.
  - Time-varying volatility for P_t and P_t^*.
  - The spread may be time-varying, s_t.
  - Unobservable variables –i.e. estimation problems: adverse selection, true price, effective spread.

- By assuming $\Delta P_t^* = u_t$ (innovation to fundamental price), we have the basic set-up to be modified: $\Delta P_t = u_t + c\, \Delta q_t$
- Hasbrouck and Ho (1987) allow for positive autocorrelation in order flow –buy (sell) orders tend to be followed by buy (sell) orders.
- Glosten and Harris (1988) add an adverse selection component of transaction costs. Glosten and Harris (1988) assume asymmetric information is carried through trade frequency. They used signed volume (X_t). It is introduced in Roll's (1984) model by: $u_t = \lambda\, X_t + \varepsilon_t$.
- Huang and Stoll (1997) in the context of Glosten and Harris (1988) use trade sign (q_t) as the carrier of asymmetric information.
- George et al. (1991) also allow for adverse selection transaction costs. They find that, when autocorrelated expected returns are omitted from the equation of market efficiency, the magnitude of the spread is downward biased.

• Hasbrouk (2005) estimates c with a Bayesian approach.

- *Bayesian approach* (via Gibbs sampler)
  - Observed data: $p_1,\ldots, p_T$
  - Unobserved data:
    - Parameters, c and σ_u
    - Latent data $q = \{q_1,\ldots,q_T\}$ and $p^* = \{p^*_1,\ldots,p^*_T\}$
- To complete the framework, need:
  1) Distributional assumptions on u_t: Normal.
  2) Priors (half-normal for c; inverted gamma for s_u)
- Posterior is $f(c, \sigma_u, q, p^* | p_1,\ldots, p_T)$

- *Gibbs Sampler*
- Basic specification is: $\Delta p_t = c\, \Delta q_t + u_t$
- Given the q_t this is a normal Bayesian regression model.
- Apply standard results.

- Nonstandard part of this model:

Given c and σ_u, construct posterior for $q_1, \ldots, q_T$.

The Gibbs sampler constructs full posterior by iteratively simulating from full conditional distributions for c, σ_u, and the q_t.

• Intuition behind estimation:

A sample price path is composed of:

- Permanent (random-walk) innovations
- Temporary c-related components (reversals, bid-ask bounce)

When we look at a price path, we try to resolve the two.

Resolution will be …

- clean when reversals are distinct: $c >> \sigma_u$
- Not clan when reversals are lost in the RW innovations: $c << \sigma_u$

• Easy extension: $c_{i,t}=\gamma_i x_{i,t}$ ($x_{i,t}$ could be latent).

Transactions Data

- Recent databases such as TAQ (Trades and Quotes), TORQ (Trades, Orders and Quotes), or our Bauer options databases give us a lot of new information.
- The databases are often tick-by-tick, all transactions of every stock are recorded.
- The transactions are discrete and not evenly spaced.
- The IID assumption fails.
- New models are created to take advantage of the data (RV, ACD)
- Discreteness must be taken into account.
- In many instances, economists aggregate or filter the data.

Buy or Sell?

- When we observe a trade, we observe:
  - P: the price at which the transaction has occurred
  - Q: the number of traded shares
- But, we do not know if the trade was buyer- or seller-initiated. (They have different information content).
- We need a model to classify trades.

- Simple Model (Mark and Ready (1992) algorithm)

  - We observe bid and ask prices: P^b and P^a. (Problem: when $P^b > P^a$.)

  - Find midpoint as $P_m = (P^b + P^a)/2$
- \- If $P > P_m$ => buyer-initiated trade
- . If $P < P_m$ => seller-initiated trade.

- Other algorithms:
  - Tick test (only trade data): If P increases, buyer initiated; if P decreases, seller initiated.
  - Lee and Ready (1991) (trade and quoted data) –proximity of P to P^b or P^a determines classification: Close to bid, buyer initiated, close to ask, seller initiated. If the trade is at P_m, the tick test is used.
  - Ellis, O'Hara, and Michaely (2000) (trade and quoted data): trades at exactly bid and ask quotes are seller-initiated or buyer-initiated; all others are categorized using the tick test.
  - Odders-White (2000) (order data): timing of the order is used as the basis for determining the trade initiator. Last order (buyer or seller) is assumed to be the trade initiator.

- Issues:
  - Liquidity-demanding trades often get price improvement. Trade direction algorithm may break. Werner (2003).
  - Orders matched without specialist. Problems with Odders-White.

Information Content of Stock Trades: Hasbrouck (1988, 1991)

- Idea: New information makes agents trade
  - Larger (measured by volume) trade (trades with 'lots' of new information) must have a larger impact on prices than smaller trades.

- Hasbrouck (1991) conducts a VAR analysis.
- Finding: There is a significant and large price impact.

Price Discovery

- Madhavan (2002): *Price discovery* is the process by which prices incorporate new information.
- Similar or identical securities often trade in multiple venues.
- *Information share*: Which market leads other markets in the price discovery process.
- Hasbrouck (1995): “The information share associated with a particular market is defined as the proportional contribution of that market's innovations to the innovation in the common efficient price.”
- Lehmann (2002): “a decomposition of the variance of innovations to the long run price.”

HUC Model - Hasbrouck (1995)

The price in security market *i* differs from the fundamental price p^* only transiently. The coefficient β is there because futures and cash markets may have a slightly different basis.

$$p_{i,t} = \beta_i p_t^* + u_t$$

The fundamental price itself follows a random walk.

$$p_t^* = p_{t-1}^* + \eta_t, \quad E(\eta_t^2) = \sigma_\eta^2$$

Error terms ξ and η can be contemporaneously and serially correlated.

$$u_t = \gamma \eta_t + e_t, \quad E(e_t e_t') = \Omega,$$

This is called an *unobserved components model* because we do not observe the efficient price directly.

Permanent Component

If we assume the individual prices are *I(1),* have a VAR(r) representation, and that markets are cointegrated, the price vector has the Engle-Granger error correction form:

$$\Delta p_t = \alpha z_{t-1} + A_1 \Delta p_{t-1} + \cdots + A_r \Delta p_{t-r} + \varepsilon_t$$

$$\underset{(N-1\times 1)}{z_{t-1}} = \begin{bmatrix} p_{1,t-1} - \beta_2 p_{2,t-1} \\ \vdots \\ p_{1,t-1} - \beta_N p_{N,t-1} \end{bmatrix}$$

Matrix of long run multipliers

$$\Psi(1) = \iota\psi' = \begin{bmatrix} \psi_1 & \cdots & \psi_N \\ \vdots & & \vdots \\ \psi_1 & \cdots & \psi_N \end{bmatrix},$$

Non-Uniqueness

In computing the long-run effects of a shock, we need to take into account contemporaneous correlation

$$\Omega = E(\varepsilon_t \varepsilon_t')$$

by taking a Choleski decomposition:

$$M = \{\{m_{ij}\}_{j=1}^{i}\}_{i=1}^{N} \text{ such that } MM' = \Omega$$

• Now, of course, we have all the same problems that the macroeconomists do. The Choleski decomposition is not unique. Papers tend to report upper bound estimates.

• An argument in favor of working directly with the structural model.

Information Shares

• Hasbrouck (1995)

$$H_j = \frac{\left[\sum_{i=1}^{n} \gamma_i m_{ij}\right]^2}{\left[\sum_{i=1}^{n} \gamma_i m_{i1}\right]^2 + \left[\sum_{i=1}^{n} \gamma_i m_{i2}\right]^2 + \cdots + (\gamma_n m_{nn})^2}.$$

• Gonzalo-Granger (1991) – used by Harris, McInish and Wood (2002))

$$GG_j = \frac{\gamma_j}{\sum_{i=1}^{N} \gamma_i}.$$

• Lehmann (2002) attempts to reconcile these. Two different forms of variance decomposition. One includes the noise from the individual markets and the other does not.

• Yan and Zivot (2005)Information Share

Impulse Response Function:

$$\frac{\partial p_{i,t+k}}{\partial \varepsilon_t}$$

Cointegration restriction:

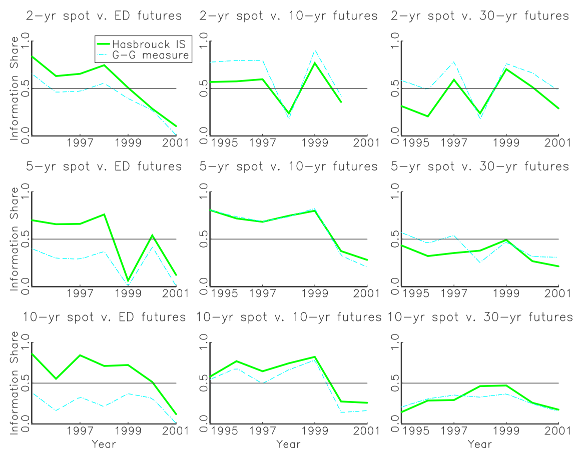
$$\frac{\partial p_{i,t+k}}{\partial \varepsilon_t} \to 1$$

Normalize with loss function to form information share:

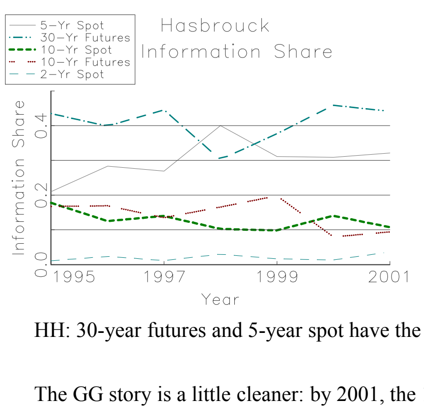
$$IS_i^{YZ} = \sum_{k=0}^{K-1} L\left(\frac{\partial p_{i,t+k}}{\partial \varepsilon_t} - 1\right),$$

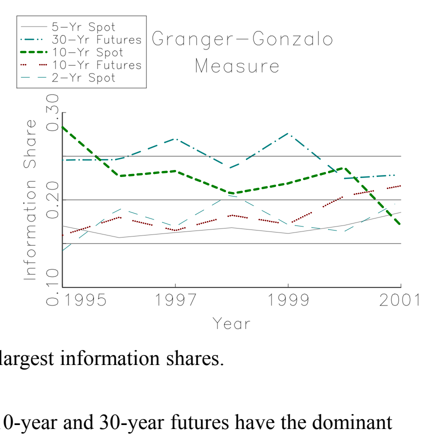
• Other estimate: deJong and Schotman (2004).

Application to U.S. Treasury Mkt: Mizrach/Neely (2006)



Full System Estimation





HH: 30-year futures and 5-year spot have the largest information shares.

The GG story is a little cleaner: by 2001, the 10-year and 30-year futures have the dominant information shares.

State Space Representation

$$p_t = Hx_t$$
$$x_t = Fx_{t-1} + v_t,$$

• For the HUC model:

$$H = \left([illegible]\; I_{N \times N} \right),\; x_t = \begin{pmatrix} p_t^{*} \\ u_t \end{pmatrix}$$

$$F = \begin{pmatrix} 1 & 0_{1 \times N} \\ 0_{N \times 1} & 0_{N \times N} \end{pmatrix},\; v_t = \begin{pmatrix} 1 & 0_{1 \times N} \\ [illegible] & I_{N \times N} \end{pmatrix} \begin{pmatrix} [illegible] \\ e_t \end{pmatrix},\; E(v_t v_t') = \begin{pmatrix} [illegible] & [illegible] \\ [illegible] & [illegible] \end{pmatrix}$$

• We are interested in estimation of the structural parameters *α*, *σ*², *Ω*. Parameters are estimated by MCMC, drawing the variance-covariance matrix of v_t and computing α, σ² and Ω using this matrix.

• We also obtain confidence measures on these estimates from the Markov chain Monte Carlo iterations. These are much less ad hoc than sample averages of daily estimates and/or the upper lower bound estimates from the Hasbrouck orthogonalization.

Information Shares – Mapping From Structural Model

Structural autocovariances:

$[illegible]_0 = E(\Delta p_t \Delta p_t') = [illegible],$
$[illegible]_1 = E(\Delta p_t \Delta p_{t-1}') = [illegible].$

Reduced form:

$\Delta p_t = [illegible] - C[illegible]_{t-1},$

$C = [illegible]$

$[illegible]_0 = [illegible] C [illegible] C',$
$[illegible]_1 = -C[illegible].$

Moments matched:

$Var([illegible]) = [illegible]$
$Cov(\Delta p_t, [illegible]) = [illegible]$
$Cov(\Delta p_t, \Delta p_{t-1}) = [illegible].$

Solution:

IS derived from these:

$[illegible]$
$[illegible]$

Structural Model Implications

- GG Information shares can be negative.
- Hasbrouck shares are positive by construction, but can give the largest IS to a market which moves prices *away* from the efficient price.
- The uncertainty of the information shares is not measured by sample average estimates of IS.

Open Questions in the Literature

Q: Does the notion of information shares make sense?

A: Without the structural model, they can be hard to interpret.

Q: Is the Hasbrouck unobserved components model (HUC) a good structural model?

A: In many ways no. Better models should exploit links to other aspects of microstructure, e.g. the bid ask spread, etc.

Conclusions

• Information shares are a useful summary statistic of the relative importance of market structures that are fragmented or where spot and derivative instruments are available.

• Despite strong identification assumptions, these measures correlate well with observable liquidity.

- Direct estimation of the structural model seems to be the best way to go forward in this literature.